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The Effects of Radiation on the Linear Stability of a horizontal layer in a Fluid-saturated Media heated from below

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ABSTRACT: The effect of radiation on the onset of Rayleigh-Benard convection is studied in the case of a radiating Newtonian fluid in a fluid-saturated horizontal porous layer heated from below. The radiative heat transfer is treated using the differential approximation for optically thin limiting case. The linear stability theory is employed to predict the onset of buoyancy-driven convective motion. It is seen that neither radiation on the static temperature nor on the disturbances can be neglected. In addition, radiation delayed the onset of instability, and higher radiation values led to greater stabilization of a gravity-driven flow in a fluid-saturated porous medium heated from below. @JASEM

The onset of Rayleigh-Benard convection in a fluid-saturated porous medium heated from below is now regarded as a classical problem. But, buoyancy-driven phenomena in porous media are actively investigated (Kim *et. al.* (2004)) due to its wide range of applications in geothermal reservoirs, agricultural product storage, enhanced oil recovery, packed-bed catalytic reactors and the pollutant transport in underground. Equally, the Rayleigh-Benard instability for radiating fluids received great attention in the past due to its implications in astrophysics and geophysical problems and for other applications such as solar collectors.

When an initially quiescent fluid-saturated porous layer is heated from below, it is well-known that the buoyancy-driven convection occurs when the Rayleigh number exceeds a certain value. Horton and Rogers (1945) and Lapwood (1948) determined the criterion for stability in terms of the critical Rayleigh number $4\pi^2$, valid for an infinitely wide horizontal porous layer. In these studies, the porous medium was assumed to be homogeneous and isotropic, and the horizontal boundaries were taken to be impermeable and perfect heat conductors. Weber (1974) and Nield (1994) studied the problem of steady convective flow which is caused by the horizontal component of the temperature gradient in a shallow horizontal layer of porous medium; while Palm *et. al.* (1972) examined the steady convection in a porous medium on the dependence of the Nusselt number on the Rayleigh number.

However, in all these studies the effect radiation has been ignored. In systems in which the operating temperatures are high, the effect of radiation becomes significant. Goody (1956) was the first to investigate effect of radiation on the Rayleigh-Benard problem in which the radiative heat transfer delayed the onset of thermoconvective instability. He considered gray media with black free boundaries and used the differential approximation for radiative heat transfers. Bdeoui and Soufiani (1997) investigated the effect theoretically the effect of radiation on the Rayleigh-Bernard instability for real gases using the integro-differential nature of the radiative transfer equation.

The aim of this present study is to investigate the effects of radiation on the linear stability of a horizontal layer in a fluid saturated porous media heated from below when the radiation (or cooling) is linearly dependent on temperature.

Basic Formulation

We consider a radiatively active horizontal fluid layer of height d heated from below and confined between two isothermal parallel surfaces at different temperatures. The lower and upper surfaces are maintained at $T_1 = T_0 + \Delta T/2$ and $T_2 = T_0 - \Delta T/2$ temperatures respectively, where $T_0 = (T_1 + T_2)/2,\ T_1 > T_2$ is the reference temperature. Assuming constant fluid properties, an exception is made for the density in buoyancy term. Taking into account the radiative heat flux in the energy equation, and assuming Darcy-Boussinesq approximation, the basic equations for convection in a homogeneous and isotropic porous media are

$$\nabla' \bullet \vec{V}' = 0 \qquad \ldots\ldots \qquad 1$$

$$\frac{\mu}{K}\vec{V}' + \nabla' P' + \rho_0 g\vec{k} = 0 \ldots\ldots \qquad 2$$

$$\rho c_p \left(\frac{\partial}{\partial t'} + \vec{V}' \bullet \nabla'\right) T' = \kappa \nabla'^2 T' - \nabla' \bullet \vec{q}_r \ldots\ldots 3$$

$$\rho_0 = \rho[1 - \beta(T' - T_0)] \qquad \ldots\ldots\ldots 4$$

where $\vec{V}' = (U', V', W')$ is the fluid velocity, μ the viscosity, g the gravitational acceleration, ρ_0 is the fluid density, c_p the specific heat capacity, κ the thermal diffusivity of the porous medium, β the

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coefficient of thermal expansion and $\vec{q}_r$ the radiative heat flux.

The boundary conditions are

$$W' = 0,\ T' = T_1 \quad \text{at } d = -d/2$$
$$W' = 0,\ T' = T_1 \quad \text{at } d = d/2 \qquad \ldots\ldots..5$$

Since the medium is optically thin with relatively low density, the radiative heat flux in the energy equation in the spirit of Cogley *et. al.* (1968) is

$$\vec{\nabla}' \bullet \vec{q}_r = 4\gamma^2 (T' - T_0),\ \gamma^2 = \int_0^\infty \delta\lambda \frac{\partial B}{\partial T'} \qquad \ldots 6$$

where δ, λ, B are the radiation absorption coefficient, frequency of radiation and Planck's constant respectively.

It is now convenient to write the governing equations in non-dimensional form as

$$\nabla \bullet \vec{V} = 0 \qquad 7$$

$$\vec{V} + \nabla P - Ra\, T\, \vec{k} = 0 \qquad 8$$

$$\left(\frac{\partial}{\partial t} + \vec{V} \bullet \nabla\right) T = \left(\nabla^2 - N^2\right) T \qquad 9$$

where we have used the following non-dimensional variables

$$(x', y', z') = d(x, y, z),\ t' = \frac{\rho c_p d^2}{\kappa} t, T = \frac{T' - T_0}{T_1 - T_2}, \vec{V}' = \frac{\kappa}{d}\vec{V},$$
$$P = \frac{K(P' + \rho g z)}{\mu\kappa}, N^2 = \frac{4\gamma^2 d^2}{\kappa},\ Ra = \frac{\rho g \beta K d}{\kappa\mu}(T_1 - T_2) \qquad ..10$$

Here Ra is the Rayleigh number and N^2 is the radiation (or cooling) parameter.

The boundary conditions now are:

$$W = 0, T = \pm\frac{1}{2} \quad \text{at} \quad z = \mp\frac{1}{2} \qquad .....11$$

Basic Flow and Linearization

The basic state of the system is given by the static solution $\vec{V} = 0$ of the governing equations (7)-(9), to which corresponds the static temperature θ_0, given in terms of the radiation (cooling) parameter, *N by*

$$\left(\frac{d^2}{dz^2} - N^2\right)\theta_0 = 0 \qquad \ldots.12$$

subject to

$$\theta_0\left(\mp\frac{1}{2}\right) = \pm\frac{1}{2} \qquad 13$$

and a pressure field p_0 satisfying $\nabla p_0 = Ra\,\theta_0 \vec{k}$.

The solution to Eq. (12) subject to conditions (13) is given by

$$\theta_0(z) = \frac{Sinh[Nz]}{2Sinh[N/2]} \qquad \ldots.14$$

By integrating Eq. (14) we obtain the static pressure

$$p_0 = \frac{Cosh[Nz]}{2NSinh[N/2]} \qquad \ldots.15$$

In order to study the stability of this static (base) state, and thus determine the onset conditions of the Rayleigh-Benard instability, we consider the perturb state define by

$$\vec{V} = \mathbf{0} + \vec{v}, T = \theta_0 + \theta, P = p_0 + p \qquad \ldots 16$$

Substituting Eq.(16) into Eqs.(7)-(9) yield the following non-dimensional equations which govern the evolution of disturbance $(\vec{v}, \theta, p)$ to the static state

$$\nabla \bullet \vec{v} = 0 \qquad 17$$

$$\vec{v} + \nabla p - Ra\,\theta \vec{k} = 0 \qquad 18$$

$$\frac{\partial \theta}{\partial t} + \alpha_s w = \left(\nabla^2 - N^2\right)\theta \qquad 19$$

where $\alpha_s \left(= -\frac{\partial \theta_0}{\partial z}\right)$ is the non-dimensional static temperature gradient.

Following the classical procedure of Chandrasekhar (1961) the system (17) –(19) is reduced to a set of two scalar equations by taking the double curl of (18) and retaining only the z-component yield

$$\nabla^2 w = Ra \nabla_h^2 \theta \qquad 20$$

$$\frac{\partial \theta}{\partial t} + \alpha_s w = \left(\nabla^2 - N^2\right)\theta \qquad 21$$

with boundary conditions

$$w = 0,\ \theta = 0 \quad \text{at} \quad z = \mp\frac{1}{2} \qquad 22a$$

$$\frac{\partial^2 w}{\partial z^2} = 0 \quad \text{on a free surface} \qquad 22b$$

where $\nabla_h^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$.

Normal Mode Analysis

Following Chandrasekhar (1961), we study the stability of normal mode disturbances, which are chosen in our case to be two dimensional waves in the horizontal plane (x, y). Each normal mode is defined by a non-dimensional wave number *k*, and the corresponding disturbances have temporal and spatial horizontal dependences of the form $\exp[i(k_x x + k_y y) + \sigma t]$, where $k^2 = k_x^2 + k_y^2$ and σ is a complex number. Hence, we search for solutions of the form

$$w(x, y, z, t) = W(z)\exp[i(k_x x + k_y y) + \sigma t],$$

$$\theta(x, yz, t) = \Theta(z)\exp[i(k_x x + k_y y) + \sigma t] \quad ..23$$

Substituting Eq.(23) into Eqs.(19) –(20) and the boundary conditions (22) and using *D* to denote the nondimensional derivative operator $\frac{\partial}{\partial z}$, we obtain

$$(D^2 - k^2)W = -Rak^2\Theta \qquad 24$$

$$(D^2 - k^2 - N^2 - \sigma)\Theta = \alpha_s W \qquad 25$$

subject to the boundary conditions

$$W = 0 = \Theta \quad \text{at } z = \pm\frac{1}{2}$$

$$\frac{\partial^2 W}{\partial z^2} = 0 \quad \text{on a free surface} \quad ....26$$

Next, we reduce Eqs. (24)-(25) into a single scalar equation by eliminating Θ. By operating on Eq.(24) with $(D^2 - k^2 - N^2 - \sigma)$ and using Eq(25), we obtain

$$[(D^2 - k^2 - N^2 - \sigma)(D^2 - k^2) + Ra\alpha_s k^2]W = 0 \quad .... 27$$

For situations of idealized free-free boundaries, Eq.(27) has solution of the form

$$W = w_0 Sin(\pi z) \quad ....28$$

for the lowest mode, where w_0 is a constant. By substituting Eq(28) into Eq(27) and simplifying yield

$$Ra = \frac{Sinh(N/2)}{(N/2)Cosh(Nz)}\frac{(\pi^2 + k^2)(\pi^2 + k^2 + N^2 + \sigma)}{k^2} \quad 29$$

For stationary convection (i.e. the onset of instability), we put $\sigma = 0$ and $Ra = Ra_s$ in Eq(29) and obtain

$$Ra_s = \frac{Sinh(N/2)}{(N/2)Cosh(Nz)}\frac{k^4 + (2\pi^2 + N^2)k^2 + \pi^2(\pi^2 + N^2)}{k^2} \quad 30$$

The critical wave number is obtained by putting $k = k_c$ and finding the minimum of $Ra_s(k_c)$. Following Chandrasekhar (1961), we minimize Eq(30) as

$$\frac{\partial Ra_s}{\partial k_c^2} = \frac{Sinh(N/2)}{(N/2)Cosh(Nz)}\frac{k_c^4 - \pi^2(\pi^2 + N^2)}{k_c^4} = 0$$

and obtain

$$k_c^2 = \pi\sqrt{(\pi^2 + N^2)} \quad .....31$$

Substituting the values of the wave number k_c from Eq(31) into Eq(30) yields the critical Rayleigh number

$$Ra_{cri} = \frac{Sinh(N/2)}{(N/2)Cosh(Nz)}[2\pi^2(\pi^2 + N^2)^{1/2} + (2\pi^2 + N^2)] \quad 32$$

RESULTS AND DISCUSSION

For the analysis of stationary convection in the presence of radiative heat transfer, the critical Rayleigh number is given in Eq.(32) with the critical wave number in Eq.(31). From Eq.(32) it is observed that Ra_{cri} is minimum when the denominator is maximum. This value occurs at $z = 0$, that is, at the centre of the horizontal channel. Thus

$$Ra_{cri}(z = 0) = [2\pi\sqrt{\pi^2 + N^2} + 2\pi^2 + N^2]\frac{Sinh(N/2)}{(N/2)} \quad 33$$

Further, for small values of the radiation parameter, $N(<< 1)$ $\frac{Sinh(N/2)}{(N/2)} \approx 1$ and so Eq.(33) reduces to

$$Ra_{cri} = [2\pi\sqrt{\pi^2 + N^2} + 2\pi^2 + N^2] \quad 34$$

In the absence of the radiation parameter, our result reduces to the onset criterion in terms of Rayleigh number determined by $Ra_{cri} = 4\pi^2$ with the wave number, $k_c = \pi$ for which the temperature gradient is just $-z$. This is in agreement with the results of Horton and Rogers (1945) and Lapwood (1948).

In order to investigate the effects of radiation, we present the behaviour of the numerical values of the critical Rayleigh number together with the critical wave number for various values of the radiation parameter in Table 1 using Eq. (33).

Table 1: Variation of radiation parameter (*N*), and dimensionless wave number (k_c) on the critical Rayleigh number

N	k_c	Ra_{cri}
0.0	3.142	39.4784
0.1	3.14239	39.5149
0.3	3.14873	39.8071
0.5	3.1613	40.3946
0.7	3.17988	41.2835
0.9	3.20416	42.4831
1.0	3.21831	43.20368

From Table 1 it is observed that increase in radiation parameter, N delays the onset of instability. Thus, the main result of this paper is that the presence of radiation, even if the variation is small, delays the onset of instability. Also, a higher radiation value is associated with greater stabilization of a gravity-driven flow in a fluid-saturated porous medium heated from below.

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