Predicting the Concentration Characteristics of Itakpe Iron Ore for cut-off Grade Estimation

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ABSTRACT: Concentration characteristics of an ore are very critical to the estimation of cut-off grade of a deposit. A mathematical model that can lead to their quick prediction will significantly enhance ore reserve estimation, mine planning and the economy of the mining venture as a whole. In this paper attempts have been made to establish a functional relationship between ore grades and concentration characteristics of Itakpe Iron ore. The Wilcoxon signed rank test has been used to establish a significant level of correspondence between actual values and values obtained from the model. The model has shown that the performance characteristics of the processing plant and the grade of the ore both play vital roles in deciding the concentration characteristics of an ore. ©JASEM

Itakpe iron ore deposit is a magnetite-hematite mineralization consisting of 14 ore layers of economic value ranging in grade predominantly from 14.8%Fe to 41%Fe with an overall average grade of 36%Fe (NIOMC Project Report; 1980, Vol. 2). It is located in Latitude 07°36'20"N and Longitude 6°18'35" E in Okeli Local Government Area of Kogi state, Nigeria (fig 1). Exploitation is by open-pit mining method, and so far, two production benches are being advanced. The iron ore is currently being processed using magnetic and gravity methods of mineral processing; and flotation plant is being constructed for future beneficiation of the iron ore. Because of the multiple grades and constantly changing cost of production and price of iron ore in the mineral market, there is constantly the need to re-evaluate the cut-off grade of Itakpe deposit. In doing this, there arises the need for huge operational expenses on laboratory analysis to establish concentration characteristics of Itakpe ore.

Thus, the significant impact of predicting concentration characteristics of an ore like tailings grade and recovery on the economies of mining outfits cannot be over – emphasized given the enormous financial cost of laboratory analysis and time required for such analysis to be ready for onward utilization in cut-off grade assessment. Hence, various experts have attempted to establish if there is a functional relationship between ore grade (feed grade) and concentration characteristics like recovery, yield, and tailings grade which are essential variables in cut – off grade estimation. If such relationship exists, this would enable experts predict future values of such characteristics based on production history of the deposits or analogous deposits.

It has been documented by various authors after a detailed analysis of Chalcedan iron ore that no such relationship exists (Promerantsev, 1990). However, Agabalian (1994), attempted to disprove this assertion by showing a relationship between ore grade and tailings grade, and between ore grade and recovery, at Abovansky open-pit iron mine. He however, recognized the impact of plant efficiency in mineral recovery. The essence of this research is to attempt the verification of a relationship (if any) between ore grades and tailings grade and between ore grade and recovery at Itakpe, using historic concentration characteristics of Itakpe ores.

MATERIALS AND METHOD

The approach in predicting the cut – off grade of Itakpe deposit is to establish the relationship between ore grade and various concentration characteristics like tailings grade and metallurgical recovery; which can be utilized in predicting the cut – off grades for the prevailing production costs and values of ores. To predict the concentration characteristics of Itakpe ore, statistical data of these characteristics for different grade have been gathered over time from Itakpe mine.

The Least Squares Method is used to analyze the above data with a view to establishing a functional relationship between them. The Least Squares Method is a mathematical tool that is used to provide a “best fit” equation to a series of data points in the sense that it is founded on the relationship that the sum of the squares of the differences between the actual data points and the points calculated for the “best fit” straight line has a minimum value; Donohue (1983). Consequently the square of the deviation of the “best fit” equation from the actual data points can be calculated as.

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Table 1 Historic concentration characteristics of Itakpe ore 1988 – 1997

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>Tailings Grade ($T_o$)</td>
<td>12.85</td>
<td>12.3</td>
<td>12.1</td>
<td>11.91</td>
<td>11.77</td>
<td>11.75</td>
<td>11.42</td>
<td>11.41</td>
<td>11.28</td>
<td>11.22</td>
</tr>
<tr>
<td>Feed Grade ($F_o$)</td>
<td>40.95</td>
<td>38.68</td>
<td>37.88</td>
<td>36.44</td>
<td>36.45</td>
<td>36.44</td>
<td>35.01</td>
<td>34.95</td>
<td>34.42</td>
<td>34.19</td>
</tr>
<tr>
<td>Recovery ($R_o$)</td>
<td>85.7</td>
<td>84.3</td>
<td>82.9</td>
<td>82.1</td>
<td>82.1</td>
<td>82.9</td>
<td>81.88</td>
<td>81.83</td>
<td>83.0</td>
<td>81.3</td>
</tr>
</tbody>
</table>


The values of the constants $a_0$ and $a_1$ can be found in such a way that a minimum deviation of “best fit” equation from the actual data points is obtained. This can be obtained by differentiating partially first with respect to $a_0$ and later, with respect to $a_1$ and equating both expressions to zero. So that:

$$
\sigma = \sum_{i=1}^{10} [y_i - P(x_i)]^2 \rightarrow \text{min}
$$

Where

$$
y_i \text{ is the value of the given data at point } i,$$

$$P(x_i), \text{ the Best fit equation which can be represented as:}$$

$$P(x_i) = a_0 + a_1 x_i$$

And

$$\frac{\partial \sigma}{\partial a_0} = \frac{\partial \sigma}{\partial a_1} = 0$$

Where $a_0$ and $a_1$ are constants, which depend on the efficiency of the processing plant and nature of ores and while $x_i$, represents the feed grade at point $i$. For a sample size of 10 we have,

$$
\sigma = \sum_{i=1}^{10} [y_i - (a_0 + a_1 x_i)]^2 \rightarrow \text{min}
$$

Opening the brackets yields,

$$\sum_{i=1}^{10} y_i = 10 a_0 + a_1 \sum_{i=1}^{10} x_i$$

$$\sum_{i=1}^{10} [y_i - (a_0 + a_1 x_i)] = 0$$

$$\sum_{i=1}^{10} x_i = \frac{10 a_0}{a_1}$$

$$\sum_{i=1}^{10} x_i^2 = \frac{10 a_0}{a_1^2}$$

$$\sum_{i=1}^{10} [y_i - (a_0 + a_1 x_i)]^2 = \frac{10 a_0}{a_1^2}$$
Predicting the Concentration

Table 2: Calculation of variables in the least squares method for tailings grade prediction

<table>
<thead>
<tr>
<th>No.</th>
<th>$x_i$</th>
<th>$y_i$</th>
<th>$x_i y_i$</th>
<th>$x_i^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>40.95</td>
<td>12.85</td>
<td>526.21</td>
<td>1676.91</td>
</tr>
<tr>
<td>2.</td>
<td>38.68</td>
<td>12.30</td>
<td>475.76</td>
<td>1496.14</td>
</tr>
<tr>
<td>3.</td>
<td>37.88</td>
<td>12.10</td>
<td>458.35</td>
<td>1434.89</td>
</tr>
<tr>
<td>4.</td>
<td>37.03</td>
<td>11.91</td>
<td>441.03</td>
<td>1371.22</td>
</tr>
<tr>
<td>5.</td>
<td>36.45</td>
<td>11.77</td>
<td>429.02</td>
<td>1328.61</td>
</tr>
<tr>
<td>6.</td>
<td>35.64</td>
<td>11.57</td>
<td>428.17</td>
<td>1327.87</td>
</tr>
<tr>
<td>7.</td>
<td>35.01</td>
<td>11.42</td>
<td>399.81</td>
<td>1225.70</td>
</tr>
<tr>
<td>8.</td>
<td>34.95</td>
<td>11.41</td>
<td>398.77</td>
<td>1221.50</td>
</tr>
<tr>
<td>9.</td>
<td>34.42</td>
<td>11.28</td>
<td>388.26</td>
<td>1184.74</td>
</tr>
<tr>
<td>10.</td>
<td>34.19</td>
<td>11.22</td>
<td>383.62</td>
<td>1168.95</td>
</tr>
</tbody>
</table>

\[
\sum_{i=1}^{10} x_i = 366 \quad \sum_{i=1}^{10} y_i = 87.62 \quad \sum_{i=1}^{10} x_i y_i = 3837.95 \quad \sum_{i=1}^{10} x_i^2 = 13566.63
\]

In order to establish a relationship between tailings grade and feed grade, the variables in the above equations are calculated as in Table 2 using the data in Table 1 where:

$\rightarrow$ Represents feed grade at various data points

$y_i$ → Represents tailings grade at various data points

\% → Number of data points.

Substituting in equations 6 & 7 yields

\[
\begin{align*}
110 &= 10a_0 + 366a_1 \\
4329 &= 366a_0 + 13436.53a_1
\end{align*}
\]

Hence

\[
\begin{align*}
16a_0 + 366a_1 &= 0 \\
366a_0 + 13436.53a_1 &= 4329
\end{align*}
\]

Resolving into a matrix equation we have

\[
\begin{bmatrix}
16 & 366 \\
366 & 13436.53
\end{bmatrix}
\begin{bmatrix}
a_0 \\
a_1
\end{bmatrix} = \begin{bmatrix}
0 \\
4329
\end{bmatrix}
\]

\[
\begin{bmatrix}
a_0 \\
a_1
\end{bmatrix} = \frac{1}{1016.08} \begin{bmatrix}
366 & -110 \\
-366 & 13436.53
\end{bmatrix} \begin{bmatrix}
0 \\
4329
\end{bmatrix} = 2.679 \\
1016.08 \\
13436.53
\end{bmatrix} = \begin{bmatrix}
2.679 \\
1016.08
\end{bmatrix}
\]

\[
y_i = 2.679 + 0.249x_i
\]

Hence $a_0 = 2.679$ 

Consequently

\[
\begin{align*}
P(x_i) &= 2.679 + 0.249x_i \\
P_g &= 2.679 + 0.249F_g
\end{align*}
\]

Similarly, in establishing a relationship between metallurgical recovery $R_m$ and feed grade $F_g$ the variables in the equation will have to be calculated. In this case:

$x_i$ → Represents feed grade at various data points

$y_i$ → Represents metallurgical recovery

\% → Number of data points (10).

Table 3: Calculation of variables in least squares method for metallurgical recovery prediction.

<table>
<thead>
<tr>
<th>No.</th>
<th>$x_i$</th>
<th>$y_i$</th>
<th>$x_i y_i$</th>
<th>$x_i^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>40.95</td>
<td>85.7</td>
<td>3509.4</td>
<td>1676.91</td>
</tr>
<tr>
<td>2.</td>
<td>38.68</td>
<td>84.3</td>
<td>3260.7</td>
<td>1496.14</td>
</tr>
<tr>
<td>3.</td>
<td>37.88</td>
<td>83.8</td>
<td>3174.3</td>
<td>1434.89</td>
</tr>
<tr>
<td>4.</td>
<td>37.03</td>
<td>82.6</td>
<td>3058.7</td>
<td>1371.22</td>
</tr>
<tr>
<td>5.</td>
<td>36.45</td>
<td>82.1</td>
<td>2992.55</td>
<td>1328.6</td>
</tr>
<tr>
<td>6.</td>
<td>35.64</td>
<td>81.83</td>
<td>2859.0</td>
<td>1221.5</td>
</tr>
<tr>
<td>7.</td>
<td>35.01</td>
<td>81.88</td>
<td>2866.7</td>
<td>1225.7</td>
</tr>
<tr>
<td>8.</td>
<td>34.95</td>
<td>81.83</td>
<td>2859.0</td>
<td>1221.5</td>
</tr>
<tr>
<td>9.</td>
<td>34.42</td>
<td>83</td>
<td>2856.86</td>
<td>1184.74</td>
</tr>
<tr>
<td>10.</td>
<td>34.19</td>
<td>81.3</td>
<td>2779.65</td>
<td>1169.0</td>
</tr>
</tbody>
</table>

\[
\sum_{i=1}^{10} x_i = 366 \quad \sum_{i=1}^{10} y_i = 87.62 \quad \sum_{i=1}^{10} x_i y_i = 3837.95 \quad \sum_{i=1}^{10} x_i^2 = 13566.63
\]
Predicting the Concentration

As previously done, substituting the values of the variables in equation 6 & 7 we have

\[ 10a_0 + 366a_1 = 829.41 \]
\[ 366a_0 + 13436.53a_1 = 30378.85 \]

Again resolving into a matrix equation we have:

\[
\begin{bmatrix}
10 & -366 \\
366 & 13436.53
\end{bmatrix}
\begin{bmatrix}
a_0 \\
a_1
\end{bmatrix}
= 
\begin{bmatrix}
-829.41 \\
-30378.85
\end{bmatrix}
\]

Or

\[
P(x_i) = 62.871 + 0.548x_i
\]

So that,

\[
R_{w_i} = 62.871 + 0.548F_i
\]

We now compare the actual values with the values computed from the model applied. We do this first for the tailings grade and then for the metallurgical recovery. In both cases using the Wilcoxon Signed Ranked Test (A.I.Arua et al 1997). For this test in both cases the hypotheses are:

\[ H_0: \text{The model values do not vary significantly from the actual values.} \]
\[ H_1: \text{The model values do vary significantly from the actual values.} \]

**Table 4: Comparison of the tailings grade values**

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual Value</td>
<td>12.85</td>
<td>12.30</td>
<td>12.10</td>
<td>11.91</td>
<td>11.77</td>
<td>11.75</td>
<td>11.42</td>
<td>11.41</td>
<td>11.28</td>
<td>11.22</td>
</tr>
<tr>
<td>(D,\text{(difference)})</td>
<td>-0.03</td>
<td>-0.01</td>
<td>-0.01</td>
<td>0.16</td>
<td>0.01</td>
<td>0.00</td>
<td>0.02</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
</tr>
<tr>
<td>Rank</td>
<td>6.5</td>
<td>2</td>
<td>2</td>
<td>9</td>
<td>2</td>
<td>4</td>
<td>6.5</td>
<td>6.5</td>
<td>6.5</td>
<td>6.5</td>
</tr>
<tr>
<td>+Rank</td>
<td>9</td>
<td>2</td>
<td>2</td>
<td>4</td>
<td>6.5</td>
<td>6.5</td>
<td>6.5</td>
<td>6.5</td>
<td>6.5</td>
<td>6.5</td>
</tr>
</tbody>
</table>

**Table 5: Comparison of the metallurgical recovery values**

<table>
<thead>
<tr>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Metallurgical Recovery</td>
<td>85.70</td>
<td>84.30</td>
<td>82.90</td>
<td>82.10</td>
<td>82.10</td>
<td>82.90</td>
<td>81.88</td>
<td>81.83</td>
<td>83.85</td>
<td>81.30</td>
</tr>
<tr>
<td>Computed Value</td>
<td>85.31</td>
<td>84.07</td>
<td>83.63</td>
<td>82.84</td>
<td>82.85</td>
<td>82.84</td>
<td>82.06</td>
<td>82.02</td>
<td>81.73</td>
<td>81.61</td>
</tr>
<tr>
<td>(D,\text{(difference)})</td>
<td>0.39</td>
<td>0.23</td>
<td>-0.73</td>
<td>-0.74</td>
<td>-0.75</td>
<td>-0.76</td>
<td>-0.18</td>
<td>-0.19</td>
<td>1.27</td>
<td>-0.31</td>
</tr>
<tr>
<td>Rank</td>
<td>6</td>
<td>4</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>+Rank</td>
<td>6</td>
<td>4</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>10</td>
<td>5</td>
</tr>
</tbody>
</table>

\[ NWOSU~JOSEPH~IFEANYI,~KINGSLEY~OKENGWU;~ALIMI~ADESOJI \]
RESULTS AND DISCUSSIONS
The relationship between the feed grade and the concentration characteristics as computed from the model is shown on table 6 and thereafter depicted on figures 2 and 3.

Table 6: Concentration characteristics as computed from the model

<table>
<thead>
<tr>
<th>Year</th>
<th>Conc. Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Feed grade (Fg)</td>
</tr>
<tr>
<td>1988</td>
<td>40.95</td>
</tr>
<tr>
<td>1989</td>
<td>38.68</td>
</tr>
<tr>
<td>1990</td>
<td>37.88</td>
</tr>
<tr>
<td>1991</td>
<td>36.44</td>
</tr>
<tr>
<td>1992</td>
<td>36.45</td>
</tr>
<tr>
<td>1993</td>
<td>36.44</td>
</tr>
<tr>
<td>1994</td>
<td>35.01</td>
</tr>
<tr>
<td>1995</td>
<td>34.95</td>
</tr>
<tr>
<td>1996</td>
<td>34.42</td>
</tr>
<tr>
<td>1997</td>
<td>34.19</td>
</tr>
</tbody>
</table>

In practice when feed grade is zero, tailings grade and recovery should be zero. However, this is not the case from the two models. The implication is that the constants represent the performance characteristics of the beneficiation plant irrespective of the grade of ore, i.e. the beneficiation plant irrespective of grade of ore will provide the performance characteristics, at least equal to the value of the constants in the two models. This confirms the important role played by the performance capabilities of processing plants.

Conclusion: The model has shown that concentration characteristics of an ore depend both on the intrinsic performance characteristics of the processing plant as well as on the ore grades although the dominant role of the former should be recognized.

REFERENCES


David A.T. Donohue (1983) Production Rate Decline Curve. Published by International Human Resources Development Corporation IHRDC Boston USA.

NIOMC Project Report; 1980, Vol. 2