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An Alternative Solution to n-Puzzle Problem

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ABSTRACT: The 3, 15 63 and n-1 puzzle problems are classical puzzle problem that has been celebrated for many years however, the conventional method for solving these problem is to use heuristic approach. Since the inception of these set of puzzle problems, researchers have made efforts to derive efficient methods of solving them. These methods require a lot of guessing that does not give a prescribed guideline to arrive at a possible solution. In this paper, an attempt was made to derive a procedure for solving n-1 puzzle problem, thereby, reducing guessing as much as possible. The derived procedure was tested on 3, 15 and 63, and the results got from the testing, were used to solven-1 puzzle problems.This paper has provided an alternative solution of how to solve n- puzzle problem, thereby reducing unnecessary guesses that were inherent in existing methods.

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The *n - puzzle* problem consist of an$m$ x m board with n numbered tiles and an empty space whereby, $n = m^2 – 1$. A number on the tile next to an empty space can be slide to make way for other numbers attempting to arrange the puzzle. The objective of arranging these numbers is to reach a goal state, which completes the proper arrangements of the set of numbers on the puzzle board. The description of the problem solving an *n - puzzle*consists of the following: *set of states*, *initial state*, *goal state*, *path cost* and *successor function*. Formally, P = (S, S_0, F, f, C): Where, S = Set of states;S_0 = Initial state;F = Final state. This can be one of the goal states already defined; f = Function called successor function generates the next state. This state can be described as a move from left to right, from up to down, etc; C = Path cost. It is the number of steps in the path of an arrangement, considering each move to be of unit cost.

This problem requires elicitation of transition rules and definite way of estimating the cost and making the heuristics. Each move in an n – puzzle problem generates a new configuration of arrangement in the puzzle board (Bhasin and Singla, 2012;Markov et al., 2006). For a period of twenty years, the 8-puzzle and 15-puzzle of Sam Loyd, have been a laboratory for testing search methods. The 15-puzzle has a long and interesting history that is said to date back to the 19 century and it continues to appear as apps on mobile devices and as mini-games inside larger video games. It consists of 15 squares numbered from 1 to 15 that are placed in a 4x4 board leaving one empty space position,without a number. The goal is to reposition the numbers from a given arbitrary starting arrangement, by sliding each one of them at a time into the configuration, which completes the arrangement.Sometime, it may not be possible to arrive at a goal arrangement from an arbitrary starting position, because of the difficulty involved in guessing the right move that will move the arrangement towards a goal arrangement(Klimaszewski, 2014;Parberry, 2014;Ratner D; Warmuth, 1990; Horowitz and Sahni, 1978; Loy, 2005; Weisstein, 2016).

Since the time when 8, 15 and N puzzle problems were created, some researchers have made efforts to either find alternative solutions to these problems or recommends improvements to existing ones. Literature of existing research works are hereby review in this section. One of the open problems of a set of puzzle problems, is determine the average number of moves used in optimal length solutions to the 15-puzzle problem. Parberry (2014), in his paper, attempted to fine an alternative solution in arranging $m^2 – 1$ numbers on the puzzle board. Experiments were conducted on 3, 15 63 and n-1 puzzle problems to understand the efficient way to arrange the numbers on the puzzle board. An algorithm was developed to formally describe the procedure of arranging $m^2 – 1$ on the nxn puzzle board. Kuruvilla and Mujahid (2014) compares the performance of popular Artificial Intelligence (AI) techniques, namely the Breadth First Search, Depth First Search, A* Search, Greedy Best First Search and the Hill Climbing Search. The authors' research work looked at the complexity of each algorithm as it tries to approach the solution, in order to evaluate the operation of each technique and identify the better functioning one in various cases. The result of their research work shows that the A* search is best suited for solving the problem of this nature. Klimaszewski (2014) presented results of the experiment, in which data structures, used by the A* algorithm (i.e. priority queue and hash table), were tested. A* algorithm was used to solve 15-puzzle, where puzzle's state was kept in the 64-bit integer variable. Bhasin and Singla (2012) presented a Genetic Algorithm (GA) based technique to solve N – Puzzle problem. The algorithm has been analyzed and it is a belief that the presented algorithm has complexity better than most of the previous research works studied. Analysis on the complexity of the solution proposed in terms of time by taking the length of chromosome as power of 2, the calculations for complexity become simpler. The advantages of the method attributed to its simplicity and unknown constants of regression equation can be easily computed.

METHODOLOGY

The proposed method of solving *n-1* puzzle problem is derived by attempting to solve *3, 15, 63* and *n-1* puzzles. The observations noticed from solving*3, 15* and *63* puzzles, will assist in deriving a procedures for solving *n-1* puzzles.

Solution to 3-puzzle Problem: Given the following 3 puzzle problem shown in Table 1, it is observed that there is only one upward shift movement, to complete the arrange of the numbers in their goal position. The symbol ø is used to indicate empty space in the tiles. From the initial arrangement positions of the numbers, the goal arrangement is achieved by shifting the number 2 upward, to occupy the second column of the first row, in the 4 tiles configuration.

Table 1: Solving 3-Puzzle Problem

Initial Arrangement		Goal Arrangement	
1	Ø	1	2
3	2	3	ø

The only movement made is the shifting of the number 2 upward, created an empty space in its former position. So, the first observation will now be: Given a 2x2 tiles, only one number (i.e., 2) is shifted upward one position and let's represent the shift as *S*. Then, $S_1=2$ is define as only a shift upward in a 3-puzzle configuration.

Solution to 15 puzzle Problem: Given the following 15 puzzle problem shown in Table 2, it is observed that there is a 4x4 tiles of 15-puzzle arrangement. It is assumed that the gamer is initially presented with an arrangement of an arbitrary the 15-puzzle problem, which looks like the one shown in T1 of Table 2. Table 2 comprises of six different arrangements that is aimed at achieving the goal arrangements of the 15- puzzle problem. The arrangements are Labeled T1 to T6. The first arrangement to the puzzle starts from T1, were it is assumed that the gamer is presented with an arbitrary scattered 15 different numbers of the puzzle. In T2, There is an attempt to create a range leader (L_1), whereby three other numbers are placed behind it. The number 12 was chosen because it will be used to lead other set of numbers horizontally, onto the last row. In T1, the set of numbers 12 is leading, are 15, 14 and 13 respectively. These set of numbers are queued up behind their leader 12 and their positions are labeled as P_1, P_2 and P_3 respectively. Then, the first leader L_1 (i.e., 12), leads the set of numbers 15, 14 and 13 onto the last row, which is seen in the configuration of T3. It was observed that $L_1$is the results of subtracting 3 from 15 (that is the maximum number in the puzzle) and the values of the led set (15, 14, 13) is the same, when 0, 1, and 2 were subtracted from P_1 P_2, P_3, respectively. In T3, L_2 leads P_1, P_2, P_3 into onto the last row of T3. The next leader L_2 (8) is chosen to lead the set of numbers P_1 (11), P_2 (10) and P_3 (9) onto the next row, after the last one. In other words, in T4, L_2 leads P_1 P_2, P_3 onto the next row, after the last. Same in T4, the leader $L_3$was created by subtracting 11 from 15 and this number is leading P_1 (7), P_2 (6) and P_3 (5) respectively. In T5, L_3 leads P_3 P_4, P_5 set of numbers to the second row and at the same time, it is rotating all the numbers in the first and second rows to their correct positions. And in T6, L_3 (4) leads other leaders namely, L_2 and $L_1$to their correct positions, resulting in a complete arrangements of the 15 puzzle configuration. In the shifting arrangement S_3=4, which indicates that the number 4 is the head leader and it will shift other 3 numbers, including itself.

Solution to 63 puzzle Problem: Given the following 63 puzzle problem shown in Table 3, the arrangement of the puzzle is achieved on an 8x8 tiles. To start the arrangement, it is assumed that the gamer may be presented with an initial arrangement of the 63-puzzle, shown in set of tiles displayed in T1. As similar to the earlier description of how to start the arrangement of 15 puzzle, a leader will be selected and will lead a set of numbers to occupy the last row of the 8x8 tiles, as shown in Figure 3.

Table 2: Solving 15-Puzzle Problem

			T1					T2					T3	
	2	4	12	ø	P_3=15-2	*13*	8	11	ø	P_3=15-5	*10*	*9*	4	ø
	10	11	1	7	P_2=15-1	*14*	?	?	?	P_2=15-4	*11*	?	?	?
	5	15	13	14	P_1=15-0	*15*	?	?	?	L_2=15-7	*8*	?	?	?
	6	3	8	9	L_1=15-3	*12*	?	?	?		13	14	15	12
			T4					**T5**					**T6**	
P_3=15-8	*7*	*6*	*5*	ø		1	2	3	ø		1	2	3	4
L_3=15-11	*4*	3	2	1	S_3=4	5	6	7	4		5	6	7	8
	9	10	11	8		9	10	11	8		9	10	11	12
	13	14	15			13	14	15	12		13	14	15	ø

The solution to the 63 puzzle starts from T2 of Figure 3, by selecting 56 as the leader for L_1. L_1 Leads P_1 (63), P_2 (62), P_3 (61), P_4 (60), P_5 (59), P_6 (58) and P_7 (57) onto the last row, as shown in T3. Again, L_2 (48) chosen to leads P_2 (55), P_3 (54), P_4 (53), P_5 (52), P_6 (51), P_7 (50) and P_8 (49) onto the next row, to the last one, as shown in T4 of Table 4. The various computations for L_2, P_2, P_3, P_4, P_5, P_6 and P_7 are shown in T3. In Tables 5 and 6, an arbitrary leader L_i can be derived whereby, i ranges from *4* to *7*. This arbitrary number was derived after observing the current arrangement of numbers in Tables 3 and 4. In concluding the arrangements of the set of numbers on the 63 puzzle, the number 8 in T10, leads other set of leaders in an upward shifting to place the leaders in their respective correct positions. In the shifting arrangement S_7=8, which indicates that the number 8 is the head leader and it will shift other 7 numbers, including itself. It is shown in Table 7. After shifting the set of leaders upward in one position, the arrangement of 63 puzzles is complete. In Table 6, after leaders L_4, L_5, L_6, and L_7 have finished leading their various set of numbers onto their respective rows, up to the second row, the configurations of how the numbers will appear on the 64 tiles are shown in T7 and T8.Interestingly, in T9 of Table 7, as the leader L_7, is leading its set of numbers onto the second row, its automatically rotates a set of numbers around the first and second row, creating a semi-arranged configuration for the first two rows. The only number that has not been positioned on the first row is the number 8. The reason why the number 8 is not yet in its position is that, it is the head of the leaders to shift the other leaders vertically onto the last column of the board.

Table 3: Solving 63-Puzzle Problem 1

			T1									T2				
26	56	18	51	9	42	10	Ø	P_7=63-6	*57*	48	55	54	53	52	51	Ø
27	63	17	43	52	38	39	62	P_6=63-5	*58*	?	?	?	?	?	?	?
25	44	19	50	58	12	11	37	P_5=63-4	*59*	?	?	?	?	?	?	?
45	57	3	20	8	41	40	61	P_4=63-3	*60*	?	?	?	?	?	?	?
28	46	2	21	7	13	16	59	P_3=63-2	*61*	?	?	?	?	?	?	?
24	29	4	49	53	31	60	34	P_2=63-1	*62*	?	?	?	?	?	?	?
23	30	48	55	6	14	33	36	P_1=63	*63*	?	?	?	?	?	?	?
1	47	5	22	54	32	15	35	L_1=63-7	*56*	?	?	?	?	?	?	?

Table 4: Solving 63-Puzzle Problem 2

				T3									T4				
P_7=63-13	*50*	*49*	40	47	46	45	44	ø	P_7=63-20	*43*	*42*	*41*	33	24	31	30	Ø
P_6=63-12	*51*	?	?	?	?	?	?	?	P_6=63-19	*44*	?	?	?	?	?	?	?
P_5=63-11	*52*	?	?	?	?	?	?	?	P_5=63-18	*45*	?	?	?	?	?	?	?
P_4=63-10	*53*	?	?	?	?	?	?	?	P_4=63-17	*46*	?	?	?	?	?	?	?
P_3=63-9	*54*	?	?	?	?	?	?	?	P_3=63-16	*47*	?	?	?	?	?	?	?
P_2=63-8	*55*	?	?	?	?	?	?	?	L_3=63-23	*40*	?	?	?	?	?	?	?
L_2=63-15	*48*	?	?	?	?	?	?	?		49	50	51	52	53	54	55	48
	57	58	59	60	61	62	63	56		57	58	59	60	61	62	63	56

Table 5: Solving 63-Puzzle Problem 3

				T5									T6				
P_7=63-27	*36*	*35*	*34*	*33*	24	31	30	ø	P_7=63-34	*29*	*28*	*27*	*26*	*25*	31	30	Ø
P_6=63-26	*37*	?	?	?	?	?	?	?	P_6=63-33	*30*	?	?	?	?	?	?	?
P_5=63-25	*38*	?	?	?	?	?	?	?	P_5=63-32	*31*	?	?	?	?	?	?	?
P_4=63-24	*39*	?	?	?	?	?	?	?	L_5=63-39	*24*	?	?	?	?	?	?	?
L_4=63-31	*32*	?	?	?	?	?	?	?		33	34	35	36	37	38	39	32
	41	42	43	44	45	46	47	40		41	42	43	44	45	46	47	40
	49	50	51	52	53	54	55	48		49	50	51	52	53	54	55	48
	57	58	59	60	61	62	63	56		57	58	59	60	61	62	63	56

Table 6: Solving 63-Puzzle Problem 4

	T7									T8							
P_7=63-41	*22*	*21*	*20*	*19*	18	17	8	ø	P_7=63-48	*15*	*14*	*13*	*12*	*11*	*10*	*9*	Ø
P_6=63-40	*23*	?	?	?	?	?	?	?	L_7=63-55	*8*	7	6	5	4	3	2	1
L_6=63-49	*16*	?	?	?	?	?	?	?		17	18	19	20	21	22	23	16
	25	26	27	28	29	30	31	24		25	26	27	28	29	30	31	24
	33	34	35	36	37	38	39	32		33	34	35	36	37	38	39	32
	41	42	43	44	45	46	47	40		41	42	43	44	45	46	47	40
	49	50	51	52	53	54	55	48		49	50	51	52	53	54	55	48
	57	58	59	60	61	62	63	56		57	58	59	60	61	62	63	56

Table 7: Solving 63-Puzzle Problem 5

	T9									T10							
Rotate1	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*Ø*	Shift Upward	1	2	3	4	5	6	7	8
Rotate2	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*8*	S_7=8	9	10	11	12	13	14	15	16
	17	18	19	20	21	22	23	16		17	18	19	20	21	22	23	24
	25	26	27	28	29	30	31	24		25	26	27	28	29	30	31	32
	33	34	35	36	37	38	39	32		33	34	35	36	37	38	39	40
	41	42	43	44	45	46	47	40		41	42	43	44	45	46	47	48
	49	50	51	52	53	54	55	48		49	50	51	52	53	54	55	56
	57	58	59	60	61	62	63	56		57	58	59	60	61	62	63	Ø

Solution to n-1 puzzle Problem: Given the following n-1 puzzle problem shown in Table 8, it is observed that there are *nxn* tiles of n-1puzzleproblem. It is assumed that the gamer can be presented with any initial arrangement of the n-1puzzle.

The initial scattered arrangement was not shown in this arrangement because, it is assumed that the initial arbitrary arrangement of n-1 puzzle can be imagine by the gamer and it has been illustrated in previous examples of this puzzles.

The attention of current example of the solution to n-1 puzzle problem, is rather on how the various leaders, lead the numbers to their positions on the tiles and the shifting procedure that completes the configuration. The first arrangement of n-1 puzzle is in T1 of Table 8. Since the trend of procedure of how to arrange 3, 15, 63 puzzles has been mastered, hence, their solutions can be generalized to solve n-1 puzzle. Relating the numbers 12 and 56 to be the different starting leaders (L_1) for 15 and 63 puzzles then, in n-1 puzzle, the starting leader will be n^2-n.

Table 8: Solving n-1Puzzle Problem 1

	T1				
$P_{n-1}=n^2-n+1$	n^2-n+1	n^2-2n	n^2-n-1	n^2-n-2	ø
$P_{n-2}=n^2-n+2$	n^2-n+2	?	?	?	?
$P_{n-3}=n^2-n+3$	n^2-n+3	?	?	?	?
…	…	?	?	?	?
$P_3= n^2-3$	n^2-3	?	?	?	?
$P_2= n^2-2$	n^2-2	?	?	?	?
$P_1= n^2-1$	n^2-1	?	?	?	?
$L_1=n^2-n$	n^2-n	?	?	?	?

Table 9: Solving n-1Puzzle Problem 2

	T2					
$P_{n-2}= n^2-2n+2$	n^2-2n+1	n^2-3n	n^2-2n-1	n^2-2n-2		Ø
$P_{n-3}= n^2-2n+3$	n^2-2n+3	?	?	?	?	?
$P_{n-4}= n^2-2n+4$	n^2-2n+4	?	?	?	?	?
…	…	?	?	?	?	?
$P_3= n^2-n-2$	n^2-n-2	?	?	?	?	?
$P_2= n^2-n-1$	n^2-n-1	?	?	?	?	?
$L_2=n^2-2n$	n^2-2n	?	?	?	?	?
	n^2-n+1	…	n^2-3	n^2-2	n^2-1	n^2-n

Table 10: Solving n-1Puzzle Problem3

	T3					
$P_{n-3}= n^2-3n-2$	n^2-3n+2	n^2-3n+1	n^2-4n	n^2-3n-1		Ø
$P_{n-4}= n^2-3n-3$	n^2-3n+3	?	?	?	?	?
$P_{n-5}= n^2-3n-4$	n^2-3n+4	?	?	?	?	?
…	…	?	?	?	?	?
$P_3= n^2-2n-1$	n^2-2n-1	?	?	?	?	?
$L_3=n^2-3n$	n^2-3n	?	?	?	?	?
	n^2-2n+1	…	n^2-n-3	n^2-n-2	n^2-n-1	n^2-2n
	n^2-n+1	…	n^2-3	n^2-2	n^2-1	n^2-n

Table 11: Solving n-1Puzzle Problem 4

	T4					
$P_{n-4}= n^2-4n+3$	n^2-4n+3	n^2-4n+2	n^2-4n+1	n^2-4n		Ø
$P_{n-5}= n^2-4n+4$	n^2-4n+4	?	?	?	?	?
$P_{n-6}= n^2-4n+5$	n^2-4n+5	?	?	?	?	?
	…	?	?	?	?	?
$L_4= n^2-4n$	n^2-4n	?	?	?	?	?
	n^2-3n+1		n^2-2n-3	n^2-2n-2	n^2-2n-1	n^2-3n
	n^2-2n+1		n^2-n-3	n^2-n-2	n^2-n-1	n^2-2n
	n^2-n+1		n^2-3	n^2-2	n^2-1	n^2-n

And, following the trend of solving earlier examples, the set of numbers which are led by $L_1$are P_1, P_2, P_3,…P_{n-4}, P_{n-3}, P_{n-2}, and P_{n-1} and their computations are n^2-1, n^2-2, n^2-3,…, n^2-n+3, n^2-n+2 and n^2-n+1 respectively. These numbers are led by n^2-n to form the last row of the n-1 puzzle. The placements of the set of led numbers and their leader (L_1) are shown in T2 of Table 9. Again in Table 9, L_2, P_2, P_2, P_3,…P_{n-4}, P_{n-3}, P_{n-2}, and P_{n-1} and their computations are n^2-2n, n^2-n-1, n^2-n-2, n^2-n-3,…, n^2-2n+3, n^2-2n+2 and n^2-2n+1 respectively. L_2 leads these set of numbers to occupy the row before the last row of the n-1 puzzle and this is shown in T3 of Table 10. Following the trend of how previous arrangement were achieved, in Table 10, L_3, P_3, P_4, P_5,…P_{n-4}, P_{n-3}, P_{n-2}, and P_{n-1} and their computations are n^2-3n, n^2-2n-1, n^2-2n-2, n^2-2n-3,…, n^2-3n+3, n^2-3n+2 and n^2-3n+1 respectively. L_1 leads these set of numbers to occupy the third row from bottom row of the n-1 puzzle and this is shown in T4 of Table 11. From Table 11, if the configuration of the set of numbers are placed in their correct positions, we can generalize an arbitrary leader L_k= n^2-kn and the set of numbers being led are n^2-n(k-1)-1,n^2-n(k-1)-2, n^2-n(k-1)-3, n^2-n(k-1)-4,…, n^2-n(k-1)+3, n^2-n(k-1)+2, n^2-n(k-1)+1.

Where k is the particular leader to lead its team of number onto a particular row. Following the trend of how to lead a group of integer numbers onto their respective correct positions and repeating the procedure over again, the semi-finished arrangements are shown in T5 of Table 12. At this point, only the first two row of the n-1 puzzle is yet to be arranged. After arriving at the last leader, which computed as L_{n-1}= n.

Table 12: Solving n-1Puzzle Problem 5

	T5				
P_{n-1}= *n-1*	*2n-1*	n+3	*n+2*	*n+1*	Ø
L_{n-1}= *n*	*n*	*4*	*3*	*2*	*1*
	2n+1	3n-3	3n-2	3n-1	2n
	…	…	…	…	…
	n^2-4n+1	n^2-3n-3	n^2-3n-2	n^2-3n-1	n^2-4n
	n^2-3n+1	n^2-2n-3	n^2-2n-2	n^2-2n-1	n^2-3n
	n^2-2n+1	n^2-n-3	n^2-n-2	n^2-n-1	n^2-2n
	n^2-n+1	n^2-3	n^2-2	n^2-1	n^2-n

Table 13: Solving n-1Puzzle Problem 6

	T5				
Rotate 1	1	2	n-2	n-1	Ø
Rotate 2	n+1	n+2	2n-2	2n-1	N
	2n+1	2n+2	3n-2	3n-1	2n
				…	…
	n^2-4n+1	n^2-4n+1	n^2-3n-2	n^2-3n-1	n^2-4n
	n^2-3n+1	n^2-3n+1	n^2-2n-2	n^2-2n-1	n^2-3n
	n^2-2n+1	n^2-2n+1	n^2-n-2	n^2-n-1	n^2-2n
	n^2-n+1	n^2-n+1	n^2-2	n^2-1	n^2-n

Table 14: Solving n-1Puzzle Problem 7

	T5				
Shift	1	2	n-2	n-1	n
upward	n+1	n+2	2n-2	2n-1	2n
	2n+1	2n+2	3n-2	3n-1	3n
				…	…
	n^2-4n+1	n^2-4n+2	n^2-3n-2	n^2-3n-1	n^2-3n
	n^2-3n+1	n^2-3n+2	n^2-2n-2	n^2-2n-1	n^2-2n
	n^2-2n+1	n^2-2n+2	n^2-n-2	n^2-n-1	n^2-n
	n^2-n+1	n^2-n+2	n^2-2	n^2-1	ø

Then, L_{n-1}= *n* will attempt to lead its own group onto the second row. The interesting aspect of the final leader's function is that, as it is leading the set of numbers onto the second row, it is indirectly rotating the semi-arranged set of numbers in their correct positions in the first and second rows. This arrangement can be seen in Table 13.And lastly, in Table 14, the leader *n* is attempting to shift the other leaders namely, *2n, 3n, 4n,* n^2-3n, n^2-2n and n^2-n upward to occupy their correct positions in the tiles. In the shifting arrangement S_{n-1}= *n*, which indicates that the number *n* is the head leader and it will shift other *n-1* numbers, including itself, upward onto the last column. When the final arrangement is achieved as shown in table 14, the goal arrangement is established.

RESULTS AND DISCUSSION

In Table 1, an alternative method for solving n-1 puzzle problem, whereby a leading number drags other set of numbers into their respective positions has been described. The proposed method was used to solve 3, 15, 63 and eventually n-1 puzzles problems. The first problem that was solved was the 3 puzzle problem. Although this problem is almost trivial because to achieve a goal arrangement, requires a gamer to rotate the set of numbers either clockwise or anticlockwise to place numbers in their respective positions. In this research, it was observed that there is an empty spot that is symbolized as ø and this symbol must be at the second row and second column of the 4 tiles board, to complete the goal arrangement. So, as a convention in this research work, ø is placed on the first row and last column in the initial arrangement and last row last column on the goal arrangement.

Solving the 15 puzzle is more challenging as compared to the solving the 3 puzzle problem and this is shown in Table 2. Experimenting with solving 15 puzzle problem, is aided in deriving the set of leaders that will help lead, in the process of leading other three numbers to the first and second rows of 15 puzzle problem. The introduction of numbers positioning as leaders, was used to arrange the numbers in the two bottom rows of the 15 puzzle problem. Another idea got from experimenting with 3 puzzle problem, is to rotate the first two row of 15 puzzle by an attempt made by L_3, to lead its set of numbers onto their respective positions. And finally, shift other leaders to their respective positions to finalize the configuration of 15 puzzle, completes the arrangement. Using the last leader to shift other leaders in upward direction to achieve a goal arrangement, is an idea got from experimenting with the 15 puzzle problem. In Tables 3 to 7, there was an attempt to solve the 63 puzzle problem. What is observed is solving 63 puzzle is the laborious aspect of crunching through the numbers by accounting for these fairly large numbers of leaders. A total of seven leaders are used before the arrangement gets to the upward shifting stage, shown in Table 7. In Tables 8 to 14, a generalized form of solving n-1 puzzle problem was described. The generalization was achieved by the careful observations noticed from solving 3, 15 and 63 puzzle problems. In addition to deriving an alternative method of solving n-1 puzzle problem, different ways of expressing an arbitrary leaders were described, However, their computations and the trends of how the variables and their coefficients relate to each other were noted. After careful observations of how to solve n-1 puzzle problem, an algorithm of how to solve n-1 puzzle problem was developed.

Generic Algorithm to solve n-1 puzzle problem
(i) Start
// Enter the value of *n* for *nxn* board to solve *n-1* puzzle problem
(ii) Define the value of *n*
//In array T_{nn}, whereby *n* represents the number of rows and column numbers
//this makes it a square matrix that is used to locate the positions of the numbers on the n-puzzle board
(iii) Declare an array T_{nn} to solve the values of nxn puzzle
//In array T_{ij}, I and j means the row and column positions that the arbitrary puzzle number can occupy.
(iv). Input the values related to the positions on the tiles of the array T_{ij}
(v) Compute the n-1 leaders as n^2-n, n^2-2n, n^2-3n, 3n, 2n and n.
(vi) Place the numbers on the rows by the leaders as followed:
n^2-n leads these numbers $n^2-1, n^2-2, n^2-3,\ldots, n^2-n+3, n^2-n+2$ and n^2-n+1 in row n
n^2-2n leads these numbers $n^2-n-1, n^2-n-2, n^2-n-3,\ldots, n^2-2n+3, n^2-2n+2$ and n^2-2n+1 in row n-1
n^2-3n leads these numbers $n^2-2n-1, n^2-2n-2, n^2-2n-3,\ldots, n^2-3n+3, n^2-3n+2$ and n^2-3n+1 in row n-2
n leads these numbers $2n-1, 2n-2, 2n-3,\ldots,n+3, n+2$ and $n+1$ in row 2
(vii) Rotate the numbers in row 1 and row 2 into their respective positions by *n* leading its set of numbers onto
row 2.
(viii) Shift the set of leaders' n, 2n, 3n, n^2-3n, n^2-2n and n^2-n upward in column n.
(xi) Stop

The behaviour of the generic algorithm is explained as followed: The algorithm starts from (i). In (ii), there is a definition of the value *n* for the array T_{nn}. In (iii), there is a declaration of array T_{nn}. In (iv), the positions were the individual numbers are placed, are entered into the array T_{ij}. In (v), the set of leaders are then computed. In (vi), these leaders are used to drag their sets numbers onto their respective rows, on the n-puzzle board. In (vii), the set of numbers on the first and second rows are rotated by the last leader n, when it is trying to lead its set of numbers onto row 2. In (vii), the set of leaders are shifted upward in the last column, to complete the arrangement of n-puzzle configuration. In (xi), the algorithm terminates.

Conclusion: In this paper, an alternative method of solving n-puzzle problem was proposed. The method was used to derive a generic algorithm, which can be implemented using a suitable programming language. In future research, there will be an attempt to implement the generic algorithm in a suitable programming language, which will be used to teach an n-puzzle player on how to arrange the puzzle.

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