



Sensitivity Analysis of Road Freight Transportation of a Mega Non-Alcoholic Beverage Industry

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ABSTRACT: Re-optimization can be very costly for gathering and obtaining more data for a particular problem, to curb this very expensive investment. Sensitivity analysis has been used in this work to determine the behaviour of input parameters of the formulated problem. The main goal of the study is to respectively provide, derive, observe, compare and discuss the sensitivity analysis of data that has been optimized using different methods of the optimal solution. The best method, saving the highest percentage of transportation cost, for the formulated problem is determined to be the North-West Corner method. This was carried out by arbitrarily assigning values to the available warehouses to determine the best possible demand and supply cases rather than the initial cases. Thus, more cases are advised to be supplied to FID from the Asejire plant for the optimum reduced value of transportation cost.

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The best application of real-life problem that has been successful over the century as numerous studies suggested in the optimization field is the transportation problem, Shradha (2017) solved the transportation problem of the Millennium Herbal Company using different methods and comparing the results yielded. Shradha gave several methods of solving the transportation problem and obtain the objective of evaluating the similarities and differences of these methods. Dantzig (1963) used the simplex method to determine the solution of transportation as the primal simplex transportation. He proposed that initial basic feasible solution for the transportation problem can be determined through the Column minima method, Row minima, Matrix minima, Vogel's approximation method and the North-West Corner Rule. For the optimal solution, he used the Modified Distribution (MODI).

Rao (2009) explained the concept of linear programming, the revised simplex method, duality in Linear Programming under which he worked on duality theorems and dual simplex method. The most interesting of part of this fourth chapter is the Decomposition Principle, Sensitivity of Postoptimality Analysis especially the Transportation Problem. Rao, however, concluded that there are two major methods of obtaining solutions for practical aspects of optimisation, firstly is the sensitivity equation using Kuhn-Tucker conditions and secondly sensitivity equation using the Concept of Feasible

Direction. Pursulai and Niittymaki (2001) worked on the functionality of the simulation program Simu+++ and Dispo+++ , developed at the Institute of Transport, Railway Construction and Operation in the University of Hanover, Germany. In their book *Mathematical Methods on Optimization in Transportation Systems* shared the book to two distinct parts, (i) Public Transport Models and (ii) General Transport Models. In the first part they carefully dealt with the prevention of delay in railway traffic by optimization and simulation, then the heuristic for scheduling buses and drivers for an ex-urban public transport computing with bus-driver dependencies. In the research work *Optimization Techniques for Transportation Problem of Three Variables*, Joshi (2013) used four methods, namely; the North-West Corner Method, the Least Cost Method, the Vogel and the MODI method, in the process of considering the optimization techniques of transportation for three variables, she vividly explain the steps to each method and the steps to determine the optimal solution and comparison between the MODI method and every other method. The book aims at getting the shortest, best and cheapest route to satisfying the demand from any destination. Latunde *et al.* (2016), Latunde and Bamigbola (2018), Latunde *et al.* (2019) and Latunde *et al.* (2020) also analysed the model design by sensitizing some model parameters in the approach of optimal control models to asset management and transportation problems. Therefore, the objective of this paper is to present the

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sensitivity analysis of road freight transportation of a mega non-alcoholic beverage industry in Nigeria.

MATERIALS AND METHODS

The methods used in the model formulation of the road freight transportation of a mega non-alcoholic beverage industry in Nigeria is Linear programming (LP). LP is applied to the problems related to the study of efficient transportation routes, that is, how efficiently the product from different sources of production is transported to the different destinations such that the total transportation cost is minimised.

Here, two major demand warehouses are considered: Asejire and Ikeja; and 11 major supply destinations are considered: FID, Akin, Oniyele, MGR, Adhex, FDR, Vero, BnB, Mimx, Nuhi and Ile-Iwe. The available data are utilised in the formulated model and solved using four different methods of solving transportation problems: North-West corner method (NWC), Least Cost method (LCM), Vogels Approximation method (VAM) and directly through a computer software called Maple. The model design is therefore analysed using Sensitivity analysis as a post optimality tool where the model designed are analysed to determine the behaviour of these different methods of solving the identified problem, thus recommend a possible improvement on different cases of demand and supply.

Model Formulation: Let’s consider a condition in which nine distinct source i.e $X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8, X_9$ and have to meet the request of also nine destinations, say $Y_1, Y_2, Y_3, Y_4, Y_5, Y_6, Y_7, Y_8, Y_9$. The gods available at each source is specified $x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9$ and the goods requested at each destinations $y_1, y_2, y_3, y_4, y_5, y_6, y_7, y_8, y_9$. The cost of moving goods from the source to destination can be represented on tables of say m_{ij} where the subscripts (1-9) indicate the cell , given the cost of moving from the source (origin) i to destination j, therefore m_{47} is the cost of moving goods from source x_4 to destination y_7 .

The Linear Programming method is a useful tool for dealing with such a problem as a transportation problem. Each source can supply a fixed number of units of products, usually called the capacity or availability, and each destination has a fixed demand, usually known as a requirement. The nature of the and its application in solving problems involving several products from sources to several destinations, this type of problem is frequently generally called “ The Transportation Problem ”.

Suppose a company has x warehouse and the number

of retailers to be y, we can only ship one product from x to y. We can build a mathematical model for the following transportation problem. For example: Consider the table 1

Table 1: Demand and Supply - Sample

	X_1	X_2	X_3	X_4	Supply
Y_1	1	2	4	4	6
Y_2	4	3	2	0	8
Y_3	0	2	2	1	10
Demand	4	6	8	6	24

Where X_i implies the supply to the warehouses and Y_j is the demand by the retailer outlets.

Let companies producing goods at different places (factories) say "m" factories, from $i=1,2,3,\dots,m$. Also, it supplies to different distributors or warehouses, we have this to be $S_i, i=1,2,3,\dots,n$. The demand from the factory reaches all requested places (say, wholesalers). The demand from the last wholesaler is the jth place, we call this D_j .

The problem of the company is to get goods from factory i and supply to the wholesaler j , the cost is c_{ij} and this transportation cost is *linear*. By formulation, if we transport a_{ij} numbers of goods from factory i to wholesaler j , then the cost is $c_{ij}a_{ij}$.

The problem is to find the minimum cost of transporting those goods. The condition that must be satisfied here is that we must meet the demand at each of the wholesalers' request and supply cannot exceed. Therefore, linearly, the cost of this program is

$$\min \sum_{i=1}^m \sum_{j=1}^n c_{ij}a_{ij} \quad (1)$$

The number of goods transported from the factory i is $\sum_{j=i}^n a_{ij}$. (2)

Meanwhile recall, a_{ij} is the good transported i to j . From the factory, you can transport any goods to any of the wholesalers $j=1,2,\dots,n$. Here (2) above is the addition of all goods supplied by the factory i from the first wholesaler to the last.

The goods cannot be more than the request to be supplied to the wholesaler, we, therefore, have it that $\sum_{j=i}^n \leq S_i \forall i = 1,2,\dots,m$. (3)

Similarly, the constraints to make sure demand is met at all wholesalers point is.

$$\sum_{i=i}^n \geq D_j \forall j = 1,2,\dots,n. \quad (4)$$

There would be excess demand if the sum of all supply is not more than the demand as such, the request from the wholesalers will be much after

supplies have been made to avoid this, therefore, have that

$$\sum_{i=1}^n D_j \leq \sum_{i=1}^m S_i, \quad (5)$$

if this is not holding, then the demand cannot be met. Hence, there must be enough possibly excess supply to be sure that demand is met.

It is also fair to assume that the quantities demand is exactly equal to the quantities supplied.

$$\sum_{i=1}^n D_j = \sum_{i=1}^m S_i, \quad (6)$$

When this happens it means the plan for transportation cost is perfect and the supply meets the wholesalers' need at every point and disposed of all goods that left the factory. Therefore, at the cost c_{ij} , m supplies for $i=1,2,3,\dots,m$, S_i and n demands D_j for

Table 1: The table representation of the problem (in thousand)

	FID	Akin	Oniyele	MGR	Adhex	FDR	Vero	BnB	Mimz	Nuhi	Ile-Iwe	Supply
Asejire	206	182	242	277	196	212	200	276	192	150	303	1320
Ikeja	180	206	235	261	177	197	212	255	200	198	295	1210
Demand	200	200	250	280	200	200	220	270	180	210	300	2530

Source: United States securities and exchange commission (2017) *Annual Report Pursuant to Section 13 or 15(D) of the Securities (The Coca-cola Company) 001-02217*

Let Y_1 = factory at Asejire and Y_2 = the factory at Ikeja.

X_{ij} = the units transported in crates from factory i to warehouse j respectively

$$i = 1,2,3 \dots m, n. \text{ and } j = 1,2,3 \dots, m, n.$$

Therefore, x_{11} represent the units transported from Asejire plant to FID warehouse, x_{12} implies to Akin up to x_{1m} which is from Asejire to Nuhi and lastly x_{1n} which is to Ile-Iwe.

Same as above, x_{21} represents the units transported from Ikeja Plant to FID warehouse, x_{22} represents the units from Ikeja to Akin up to x_{2m} which is from Ikeja to Nuhi and lastly from Ikeja is x_{2n} which is to Ile-Iwe.

With the knowledge of table 1, the 12 months transportation cost can be considered as:

$$\begin{aligned} \text{Min } Z = & 206x_{11} + 182x_{12} + 242x_{13} + 277x_{14} + \\ & 196x_{15} + 212x_{16} + 200x_{17} + 276x_{18} + 192x_{19} + \\ & 150x_{1m} + 303x_{1n} + 180x_{21} + 206x_{22} + 235x_{23} + \\ & 261x_{24} + 177x_{25} + 197x_{26} + 212x_{27} + 255x_{28} + \\ & 200x_{29} + 198x_{2m} + 295x_{2n} \end{aligned}$$

Subject to:

The available supply constraint is given by

$$\begin{aligned} x_{11} + x_{12} + x_{13} + x_{14} + x_{15} + x_{16} + x_{17} + x_{18} + x_{19} + \\ x_{1m} + x_{1n} \leq 1320 \\ x_{21} + x_{22} + x_{23} + x_{24} + x_{25} + x_{25} + x_{27} + x_{28} + x_{29} + \end{aligned}$$

$$j=1,2,3,\dots,n.$$

The major work is finding a transportation schedule denoted by x_{ij} to get a solution to

$$\min \sum_{i=1}^m \sum_{j=1}^n x_{ij}c_{ij} \quad (7)$$

subject to

$$\sum_{j=1}^n x_{ij} = S_i \forall i = 1,2,\dots,m. \quad (8)$$

and also to

$$\sum_{i=1}^m x_{ij} = D_j \forall j = 1,2,\dots,n. \quad (9)$$

Optimal Solutions to the Problem: This subsection shows the data gathered, the result of the optimal feasible solution obtained and the sensitivity analysis of the parameters from two major plants, namely: The Asejire and the Ikeja plant.

$$x_{2m} + x_{2n} \leq 1210$$

Likewise the demand constraint is as computed below:

$$x_{11} + x_{21} \leq 200$$

$$x_{12} + x_{22} \leq 200$$

$$x_{13} + x_{23} \leq 250$$

$$x_{14} + x_{24} \leq 280$$

$$x_{15} + x_{25} \leq 200$$

$$x_{16} + x_{26} \leq 220$$

$$x_{17} + x_{27} \leq 220$$

$$x_{18} + x_{28} \leq 270$$

$$x_{19} + x_{29} \leq 180$$

$$x_{1m} + x_{2m} \leq 210$$

$$x_{1n} + x_{2n} \leq 300$$

$$x_{1i}, x_{2i} > 0$$

$$\forall i = 1,2. j = 1,2 \dots m,n.$$

Table 2: The table representation of methods of solution and cost value (in thousand)

Methods	Cost Value
North-West Corner Method (NWC)	517,040
Least Cost Method (LCM)	535,690
Vogel's Approximation Method (VAM)	525,690
Maple	546,919

$$RCV\% = \frac{\text{Initial Cost} - \text{Optimized Cost}}{\text{Initial Cost}} \times 100 \quad (10)$$

Where RCV = The reduced cost of value

For the initial transportation cost of delivering goods to with in respect to demand, we have

$$\begin{aligned} & (206 \times 200) + (206 \times 200) + (250 \times 235) + \\ & (280 \times 277) + (200 \times 196) + (220 \times 212) + \\ & (220 \times 212) + (270 \times 276) + (210 \times 198) + \end{aligned}$$

$$(255 \times 270) + (300 \times 303) =$$

$$41,200 + 41,200 + 58,750 + 77,560 + 39,200 + 46,640 + 46,640 + 74,520 + 36,000 + 41,580 + 90,900 = 594,190.$$

For NWC, the reduced cost is

$$\frac{594,190 - 517,040}{594,190} \times 100\% = 12.98\%$$

For LCM, the reduced cost is

$$\frac{594,190 - 535,690}{594,190} \times 100\% = 9.85\%$$

For VAM, the reduced cost is

$$\frac{594,190 - 525,690}{594,190} \times 100\% = 11.52\%$$

For MAPLE, the reduced cost is

$$\frac{594,190 - 546,919}{594,190} \times 100\% = 7.96\%$$

The problem was solved with three distinct methods namely; the North-West Corner Method, The Least Cost Method and the Vogel's Approximation and then compared with the result computed by the linear programming module called MAPLE.

The study of the graph and the chart above in Figure 1 shows that the North-West Corner Method produces the optimum transportation cost which is 517,040.

Sensitivity Analysis: Using equation 10, For the initial transportation cost of delivering goods to with in respect to demand, we have

$$(206 \times 200) + (206 \times 200) + (250 \times 235) + (280 \times 277) + (200 \times 196) + (220 \times 212) + (220 \times 212) + (270 \times 276) + (210 \times 198) + (255 \times 270) + (300 \times 303) =$$

$$41,200 + 41,200 + 58,750 + 77,560 + 39,200 + 46,640 + 46,640 + 74,520 + 36,000 + 41,580 + 90,900 = 594,190.$$

For NWC, the reduced cost is

$$\frac{594,190 - 517,040}{594,190} \times 100\% = 12.98\%$$

For LCM, the reduced cost is

$$\frac{594,190 - 535,690}{594,190} \times 100\% = 9.85\%$$

For VAM, the reduced cost is

$$\frac{594,190 - 525,690}{594,190} \times 100\% = 11.52\%$$

For Maple, the reduced cost is

$$\frac{594,190 - 546,919}{594,190} \times 100\% = 7.96\%$$

The Coca-cola plc problem was solved with three distinct methods namely; the North-West Corner Method, The Least Cost Method and the Vogel's Approximation and then compared with the result computed by the linear programming module called MAPLE.

The result from Table 2 above shows that the North-West Corner Method produces the optimum transportation cost which is 517,040.

For the analysis here, we increase the number of cases in demanded by each warehouse from both plants (Asejire and Ikeja), we do this by adding 50 cases each. We run every addition by the Maple software to determine the outcome and the optimized cost that will be generated, it was studied that some warehouses had more cost value than the others. In Table 3 above X_{11} is the amount of cases demanded by FID from the Asejire plant, X_{12} is the amount of cases demanded by Akin also from the Asejire plant, it continues as represented in Table 2, Table 3 and Table 4 and continues on the row until X_{1m} which is the number of cases demanded by Nuhi and lastly X_{1n} which is Ile-Iwe. X_{21} is the amount of cases demanded by FID from the Ikeja plant, X_{22} is the amount of cases demanded by Akin also from the Ikeja plant, it continues as represented in Table 2, Table 3 and Table 4 and continues on the row until X_{1m} which is the number of cases demanded from Ikeja by Nuhi and lastly X_{1n} which is Ile-Iwe.

RESULTS AND DISCUSSION

The problem was solved with three distinct methods namely; the North-West Corner Method, The Least Cost Method and the Vogel's Approximation and then compared with the result computed by the linear programming module called MAPLE. The study of the graph and the chart above in Figure 1 shows that the North-West Corner Method produces the optimum transportation cost which is 517,040. It is noted that Optimized Result is the cost computed by MAPLE and RCV is the Reduced Cost Value in percentage. The result of the Sensitivity Analysis studied shows that more cases of drinks can be should be supplied to X_{11} which is the FID warehouse from Asejire as it has the largest optimum reduced cost value almost twice of the others. The implication of this is that by priority of supply, X_{11} will get more cases and will still minimize cost.

Table 3: The Result of the Sensitivity Analysis from the Maple

S/N	Warehouse	No of Old Cases	No of New Cases	Optimized Result	RCV %
1	X_{11}	206	256	506,420	14.77
2	X_{12}	182	232	556,920	6.27
3	X_{13}	242	292	559,420	5.85
4	X_{14}	277	327	560,920	5.60
5	X_{15}	196	246	557,920	6.10
6	X_{16}	212	262	557,920	6.10
7	X_{17}	200	250	560,420	5.68
8	X_{18}	276	326	555,920	6.44
9	X_{19}	192	242	557,420	6.19
10	X_{1m}	150	200	561,920	5.43
11	X_{1n}	303	353	597,420	-0.54
12	X_{21}	180	230	546,920	7.96
13	X_{22}	206	256	546,920	7.96
14	X_{23}	235	285	546,920	7.96
15	X_{24}	261	301	556,920	6.27
16	X_{25}	177	257	546,920	7.96
17	X_{26}	197	247	546,920	7.96
18	X_{27}	212	262	546,920	7.96
19	X_{28}	255	305	546,920	7.96
20	X_{29}	200	250	546,920	7.96
21	X_{2m}	198	248	546,920	7.96
22	X_{2n}	295	345	546,920	7.96

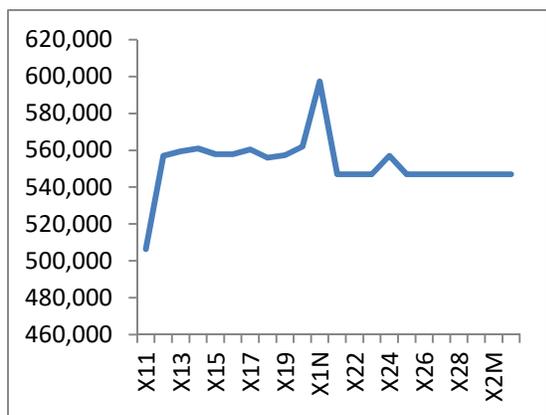


Fig 1: The Graph of Optimized Results against the number of warehouses

In Figure 1 above, we studied that X_{11} has the least Optimized Result and X_{1n} has the highest result.

We studied Figure 2 above to mean that X_{11} has the highest percentage of reduced cost value and X_{1n} has the least, this can be interpreted that the higher the Optimized Result, the lesser the Reduced Cost Value and vice-versa.

Conclusion: Since the transportation problem is one of the major problems in the Optimization field, Operation Research and even life problems to man companies. The transportation problem was formulated as a Linear Programming and solved with MAPLE software. The computational results provided the minimal total transportation cost and the values that will optimize the cost of supplying, the number of cases to supply and where to supply more cases. The

study shows that the best method that will save the highest percentage of transportation cost for this problem in the North-West Corner Method. It will save 12.98%. Also, more cases are advised to be supplied to FID from the Asejire plant for the optimum reduced value of transportation cost. Supplying 50 extra cases to FID more than other warehouses will reduce by 14.77%

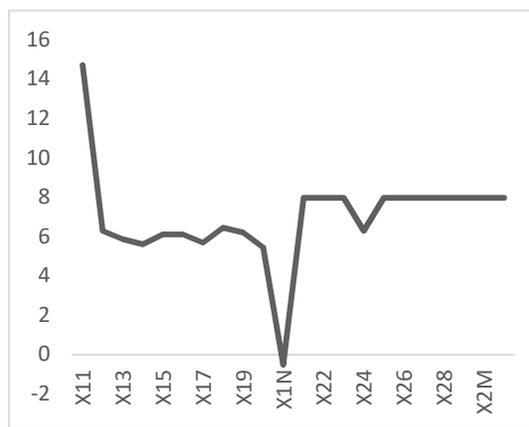


Fig 2: The Graph of Reduced Cost Value against the number of warehouses

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